**Practice Problems (Part II)** **Name**

*Please remember: You may use your calculator only in elementary and trig modes but not in calculus mode.*

1. Of the numbers listed below, the one that approximates the area of the region bounded by the graph of $y = \ln x$, the $x$-axis, and the lines $x = 1$ and $x = 5$ best is:

   A. 2.5 B. 3 C. 3.5 D. 4 E. 4.5

2. The value of the integral $\int_0^{\frac{\pi}{2}} x\,(\sin \frac{x}{2})\,dx$ is:

   A. $2\sqrt{2} - \pi$ B. $2\sqrt{2} - \frac{\pi}{2}\sqrt{2}$ C. $\sqrt{2} - \sqrt{2}\pi$ D. $2\sqrt{2} - \sqrt{2}\pi$ E. $\frac{2}{\sqrt{2}} - \frac{\sqrt{2}}{2}\pi$

3. Let $y = f(x)$ be a solution of the differential equation $x\frac{dy}{dx} = 2y + x^3 \cos x$ (where $x > 0$) that satisfies $f(\pi) = 1$. Then $f(\frac{\pi}{2})$ is equal to:

   A. $\frac{1}{4} - (\frac{2}{\pi})^2$ B. $\frac{2}{\pi} + \pi$ C. $\frac{1}{\pi} + 2$ D. $1 - (\frac{2}{\pi})^2$ E. $(\frac{\pi}{2})^2 + \frac{1}{4}$

4. Let $y = f(x)$ be a solution of the differential equation $y\,\frac{dy}{dx} = xe^{x^2}$ that satisfies $f(x) > 0$ and $f(0) = 0$. Then $f(1)$ is equal to:

   A. $\sqrt{e-1}$ B. $\sqrt{e+1}$ C. $\sqrt{e^2-1}$ D. $\sqrt{2e^2+1}$ E. $\sqrt{e^2+1}$

5. Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 20y = 0$. Then find the particular solution $y = f(x)$ that satisfies $f(0) = 0$ and $f'(0) = 4$. The value of $f(\frac{\pi}{8})$ is

   A. $2e^{\frac{\pi}{4}}$ B. $2e^{\frac{\pi}{4}}(1 + \frac{\sqrt{2}}{2})$ C. $e^{\frac{\pi}{4}}$ D. $3e^{\frac{\pi}{4}}(1 + \frac{\sqrt{2}}{2})$ E. $e^{\frac{\pi}{2}}$

6. You are given a complex plane with a polar coordinate system. Take the numbers $c_1 = (2, \frac{1}{4}\pi)$, $c_2 = (5, \frac{9}{4}\pi)$, and $c_3 = (-5, \pi)$. The number $c_1c_2 + c_3$ is equal to

   A. $-5 + 10i$ B. $10$ C. $10 - 5i$ D. $10 + 5i$ E. $5 + 10i$

7. You are given a plane with a polar coordinate system and the equation $r = \frac{5}{1+3\cos\theta}$. Because $3 > 1$ its graph is a hyperbola. The box in the polar plane that determines the graph of this hyperbola has center (in polar coordinates) and dimensions given respectively by:

   A. $(0, \frac{15}{8})$; $\frac{5}{8}\times\frac{5}{\sqrt{8}}$ B. $(\frac{5}{8}, 0)$; $\frac{5}{8}\times\frac{5}{\sqrt{8}}$ C. $(-\frac{15}{8}, 0)$; $\frac{5}{8}\times\frac{5}{\sqrt{8}}$ D. $(\frac{15}{8}, 0)$; $\frac{5}{8}\times\frac{5}{8}$ E. $(\frac{15}{8}, 0)$; $\frac{5}{8}\times\frac{5}{\sqrt{8}}$

   Problems 8 and 9 deal with a polar plane and the spiral $r = f(\theta) = \theta^{\frac{3}{2}}$ where $\theta \geq 0$. Any angle $\theta$ determines a ray in the polar plane starting at the origin $O$. For $\theta = 0$, let $P$ be the origin $O$. For any $\theta > 0$ let $P \neq O$ be the first point of intersection of the ray and the spiral.

8. Rotate the ray from $\theta = 0$ to $\theta = 2\pi$. The part of the area that the segment $OP$ traces out that falls outside the circle of radius 1 is equal to:

   A. $2\pi^4$ B. $2\pi^4 - \pi$ C. $2\pi^4 - \pi + \frac{1}{2}$ D. $2\pi^4 - \pi + \frac{3}{8}$ E. $2\pi^4 - \pi^2$

9. As the segment $OP$ rotates from $\theta = \frac{\pi}{2}$ to $\theta = 2\pi$, the positive angle $\alpha$ that the tangent to the spiral at $P$ makes with $OP$:

A. increases by about 30° B. decreases by about 15° C. decreases by about 45°
D. increases by about 45° E. decreases by about 30°

Problems 10 and 11 deal with the graph of the polar equation $r = f(\theta) = \dfrac{6}{3\sin\theta + 2\cos\theta}$.

10. The integral $\displaystyle\int_0^{\frac{3\pi}{4}} \tfrac{1}{2} f(\theta)^2 \, d\theta$ is equal to

A. 6 B. 7 C. 8 D. 9 E. 10

11. The integral $\displaystyle\int_0^{\frac{3\pi}{4}} \sqrt{f(\theta)^2 + f'(\theta)^2} \, d\theta$

A. $\sqrt{113}$ B. $\sqrt{115}$ C. $\sqrt{117}$ D. $\sqrt{119}$ E. $\sqrt{121} = 11$

Formulas and expressions: $\int u\,dv = uv - \int v\,du$ $\quad Ay'' + By' + Cy = 0$ $\quad y = D_1 e^{r_1 x} + D_2 e^{r_2 x}$

$y = D_1 e^{2x} + D_2 x e^{2x}$ $\quad y = e^{ax}(D_1 \cos bx + D_2 \sin bx)$ $\quad e^{i\theta} = \cos\theta + i\sin\theta$ $\quad x = r\cos\theta$ $\quad y = r\sin\theta$

$a = \frac{d}{1-\varepsilon^2}$ $\quad b = \frac{d}{\sqrt{1-\varepsilon^2}}$ $\quad a = \frac{d}{\varepsilon^2-1}$ $\quad b = \frac{d}{\sqrt{\varepsilon^2-1}}$ $\quad f'(\theta) = f(\theta)\cdot\tan(\alpha - \frac{\pi}{2})$ $\quad \int_a^b \sqrt{f(\theta)^2 + f'(\theta)^2}\,d\theta$

$\frac{1}{2}r^2\theta$ $\quad \int_a^b \frac{1}{2}\,f(\theta)^2\,d\theta$ $\quad \frac{d\theta}{dt} = \frac{2\kappa}{r^2}$ $\quad 2\frac{dr}{dt}\cdot\frac{d\theta}{dt} + r\cdot\frac{d^2\theta}{dt^2} = 0$